A Tribute to Tau

Apostolos P. Georgopoulos

Brain Sciences Center, Veterans Affairs Medical Center, Minneapolis, USA
Department of Neuroscience, University of Minnesota Medical School, Minneapolis, USA
Department of Neurology, University of Minnesota Medical School, Minneapolis, USA
Department of Psychiatry, University of Minnesota Medical School, Minneapolis, USA
Center for Cognitive Sciences, University of Minnesota, Minneapolis, USA

I love \( \tau \) because it's a good-looking letter of the Greek alphabet. I also love \( \tau \) because it was invented by my good friend Dave N. Lee, long before we met. In fact, it was very nice of him to immortalize this measure, presciently, using Greek in 1976 (Lee, 1976), some 12-odd years before our acquaintance was made in 1984 in Bielefeld, Germany. At that time, we spent the whole night musing over \( \tau \) with Claes Von Hofsten and Dave over a bottle of decent single malt. That night also clinched a lifelong friendship with Dave and Claes, and an ongoing collaboration with Dave for the past 15 years.

All truth be told, \( \tau \) captures perhaps the most fundamental aspect of goal-oriented behavior, and, quite possibly, all behavior. But, to appreciate its value, you have to rethink behavior as an evolving set of interlocking, goal-oriented actions. This is essential, for the instantaneous \( \tau \) defines that unique time-to-completion period, given your current, instantaneous speed, \( p \), toward the goal and the current, instantaneous amplitude of the gap, \( D \):
The "completion" is your endpoint, be it hitting a wall, delivering a grant application, landing an aircraft, or any discrete event at which you are aiming. In other words, think of all behavior as a set of gaps-to-be-closed, or goals-to-be-met, by a certain time, that is, by a deadline. Then you have to worry about closing the gap on time, coordinating multiple, associated gap closings, and taking into account your own gaps, those of others, and the association among those meaningfully related (Lee, 1998). The temporal aspect is the crucial element in the business: not the absolute time, of course, but the interval of time from the current instant to the moment of closure. On the other hand, it should be noticed that all variables in Equation 1 are instantaneous estimates, and, hence, functions of time. From this it follows that the shape of the function \( \tau(t) \) can be arbitrary, depending on the time-varying shape of \( p \) and \( D \). In the special case where the speed is constant, the goal fixed, and the path-to-the-goal straight, it has been found that the shape of \( \tau(t) \) is very similar across various sensory and motor behaviors, including vision, echolocation, reaching, flying, and so forth, and in different species, including humans, birds and bats (Lee, 1998). This led to the hypothesis that the temporal evolution of diverse biological finite processes (i.e., with beginning and end) is governed by the same "generalized" \( \tau(t) \) function, called \( \tau \)-guide (\( \tau_g \)) (Lee, 1998), of the form

\[
\tau_g(t) = 0.5(t - T^2/t)
\]  

(2)

where \( T \) is the duration of the movement and \( t \) is the time from the start of the movement. This equation implies a process evolving with constant acceleration, and is derived directly from Newton’s equations of motion under constant force, for example, gravity. When dealing with multiple processes, coupling of separate \( \tau(t) \) functions has been proposed (Lee, 1998; Lee, Craig & Grealy, 1999; Lee, Georgopoulos, Clark, Craig, & Port, 2001) as a mechanism for temporal coordination of the ensemble of processes:

\[
\tau'(t) = k \tau^B(t)
\]  

(3)

where \( A \) and \( B \) denote two different processes, and \( k \) is a gain constant.

4. A TRIBUTE TO TAU

This generalizes the approach to multiple dimensions and can be extended without difficulty to sequential processes.

So far, so good for the limiting case of constant speed, fixed goal, and straight gap. What if these variables themselves change with time? What if your speed changes arbitrarily, the target is moving, and the path of the gap is tortuous? The \( \tau(t) \) is still of importance for successful closure but now the organism has to work with slippery estimates (confidence intervals, so to speak) rather than solid data. Consider trying to hit a fly on-the-fly, or running to hit a tennisball, or running to catch an escaping thief. In such cases, the instantaneous gap, \( D(t) \), is uncertain, and solid knowledge is replaced by an educated guess. In fact, we all know how important a fortunate prediction of the target’s instantaneous position, and hence of \( D(t) \), is for successful closure. Obviously, the situation can become quite complicated when dealing with coupling multiple, uncertain \( \tau'(t) \), \( \tau^A(t) \), \( \tau^C(t) \),... where correct educated guessing in many fronts can make the difference. Formally, in these cases, \( \tau(t) \) is replaced by a confidence interval \( \{\tau(t)^{LOW}, \tau(t)^{HIGH}\} \), which might change as conditions change over time. This statistical perspective on \( \tau(t) \) is closer to reality. For, in any realistic application, \( D(t) \) and \( p(t) \) are always estimates, that is, they are never known with absolute certainty; and the same holds for the coupling coefficient \( k \) in Equations 3 and 4. This statistical variation stemming from the uncertainty in estimation of \( \tau(t) \) should also explain part of the variation in the success of closing the gap. A thorough treatment of this generalized, statistical extension of \( \tau(t) \) and its couplings remains to be worked out.

How are brain events related to \( \tau \) ? Because brain processes underlie all behavior, in principle, one need only substitute the set of neural events for the corresponding behavioral variables to derive the neural representation of \( \tau(t) \). More specifically, I refer to the neural representation of \( D(t) \) and \( p(t) \), say, \( N^{p(t)} \) and \( N^{D(t)} \). Then the neural representation of \( \tau(t) \), \( N^{\tau(t)} \), is given by

\[
N^{\tau(t)} = N^{D(t)} / N^{p(t)}
\]  

(5)
Although our knowledge about \( N_D^o \) and \( N_p^o \) is currently incomplete, this is an empirical issue which, in due time, will be elucidated. A different question concerns whether there may be a direct representation of \( \tau(t) \) in the brain such that brain events are shown to be related to \( \tau(t) \) directly, and not through the intermediate (component) representations of \( N_D^o \) and \( N_p^o \). Two approaches have been employed to investigate this problem. In one, the time course of neural activity in the motor cortex (MC) and parietal cortex (PC) of behaving monkeys during reaching to a target have been used to calculate \( MC_N^o \) and \( PC_N^o \), respectively, and to relate these putative neural representations of \( \tau(t) \) to the \( \tau(t) \) derived from the time course of the movement itself and the \( \tau \)-guide, \( \tau_g(t) \) (Lee, et al., 2006). In the other approach, the hypothesis was tested that the activity of single cells in the posterior parietal and motor cortex would be related to \( \tau(t) \) whereas monkeys intercepted stimuli moving in real or path-guided apparent motion along a circle and at various speeds (Merchant, Battaglia-Mayer, & Georgopoulos, 2004). Both approaches provided rich insights into the potential neural representation of \( \tau(t) \) both at the single cell (Merchant et al. 2004) and the population level (Lee, et al., 2006).

These considerations bring forth a crucial element underlying all applications of \( \tau \) and its neural representations, namely the prospective, goal-directed nature of behavior. The world of \( \tau \) is inhabited by prospective closures. To paraphrase William James who said that “we are a bundle of walking habits” (James, 1890, p. 127), we can say that “we are a bundle of prospective gap closures.” How could it be otherwise? The question, of course, is not whether this statement is true (yes, it is) but whether this way of looking at things is indispensable for understanding behavior. All kinds of behaviors have been described and analyzed for many years without reference to the \( \tau \) concept, and, maybe, this is why we still understand so little. The \( \tau \) gospel states that there can be no understanding of behavior outside gap-closings; of meaningful, goal-directed behavior, that is. This point is critical, as is the closely associated concept of prospective control. If future is not in the picture, the \( \tau \) measure is out of the picture too. How reasonable is the assertion of the importance of taking into account the future for current behavior? In Plans and the Structure of Behavior, Miller, Galanter and Pribram (1960) advanced persuasive arguments for prospective planning and control in behavior. In a way, \( \tau \) theory can be regarded as the requisite elaboration of these points in the temporal domain. As discussed previously (Georgopoulos, 2002), \( \tau \) theory is not concerned with the content of the plans or the nature of the specific behavior, it deals only with meeting goals on time. In that sense, the \( \tau \) function can be regarded as a general, formal procedure operating on various “contents” (e.g., behaviors) without carrying case-specific information by itself. And this makes Dave’s contribution so truly fundamental.

ACKNOWLEDGMENTS

This work was supported by the Carnegie Trust for the Universities of Scotland, the United States Public Health Service grant NS1/413, the United States Department of Veterans Affairs, and the American Legion Brain Sciences Chair.

REFERENCES


